

Gradient and Scattering Forces on a Kerr Nanosphere

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ABSTRACT

A theoretical model that computes both for the gradient and scattering forces being exerted on a nonresonant nonlinear (electro-optic Kerr effect) rigid nanosphere by a strongly focused continuous-wave laser beam is presented. The incident wavelength of the laser beam is assumed to be appreciably larger than the nanosphere radius a . Optical forces arising from the aforesaid interaction can be derived by a two-component approach which determines individually the gradient force and scattering force. The behavior of the trapping (gradient) force is plotted against several experimental parameters, e.g., incident beam power, axial distance, sphere radius, wavelength, and refractive index difference between the surrounding liquid and the nanosphere. Results have shown that the Kerr effect on the nanosphere can produce a maximum of tenfold increase in the trapping force.

INTRODUCTION

The great advantage of using a single beam is that it can be used as an *optical tweezer* to manipulate small particles under typical inverted microscopes. Optical tweezing has found its way to life science applications particularly in cell manipulations and single-molecule diagnostics. Bockelmann et al. (2002) have studied mechanical properties of DNA by unzipping short hairpin loops in an optical trap. Much effort has been made to explain the nature of optical forces and describe them quantitatively through sound theoretical models.

We have studied the behavior of the (time-averaged) radiation that is exerted by a tightly focused cw Gaussian beam on a nonlinear microsphere both in the geometrical regime (Pobre & Saloma, 1997) ($\alpha=2\pi a/\lambda > 100$) and in the Mie regime (Pobre & Saloma, 2002) ($\alpha \leq 100$). But to our knowledge, this problem has not been addressed before for the case when the sphere

radius a is appreciably smaller than the incident wavelength λ ($\alpha \ll 1$) or in the Rayleigh regime. From an academic standpoint, the problem is worth pursuing for it involves finding significant behaviors that could accurately account for nonlinear effect, in particular Kerr effect, on the optical trapping force.

THEORETICAL FRAMEWORK

Consider a nonresonant and nonmagnetic Kerr nanosphere whose orientation with the incident Gaussian beam can be shown in three-dimensional Cartesian axes as illustrated in Fig. 1.

The nonlinear nanosphere is illuminated by a linearly polarized Gaussian beam of the fundamental mode with beam radius w_0 at its beam waist position propagating from the negative z to the positive z direction whose refractive index given by $n_2 = n_2^{(0)} + n_2^{(1)}I(R)$, where $n_2^{(0)}$

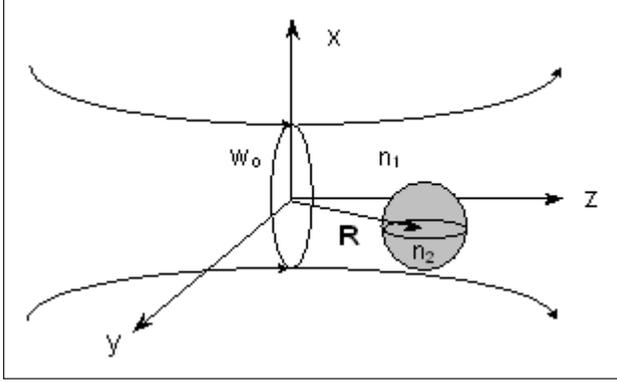


Fig. 1. Orientation of the microsphere with the Gaussian beam in the 3D cartesian axes.

is the linear component of the refractive index, $n_2^{(1)}$ is the nonlinear component, and $I(R)$ is the intensity of the Gaussian beam. The reference system of the nanosphere is in the center of the Cartesian coordinate system, which is located at the beam waist center. When the beam is incident on the nanosphere, the nanosphere in the Rayleigh scattering regime behaves like a particle that acts as a simple point electric dipole. The total radiation force, as shown in Fig. 2, acting on the dipole at r_1 for $+q$ and r_2 for $-q$ can be expressed as

$$F = F_q(r_1) + F_{-q}(r_2). \quad (1)$$

Since the particle is very small compared with the beam width, incident electromagnetic (EM) fields can be treated almost constant within the distance $|r|$. Moreover, the distance $|r|$ is much smaller than the incident wavelength, Eq. (1) can be expanded in r using a power series. If we only consider limited terms in the first order of $|r|$, we can obtain the following equations from Eq.(1):

$$\begin{aligned} F &= (\mu \cdot \nabla) E(R, t) + \dot{\mu} \times B(R, t) + R, t) + \dot{R} \times (\mu \cdot \nabla) B(R, t) \\ &= F_{\text{in, lin}}^{\text{in}} + F_{\text{Lorentz}}^{\text{Lorentz}} + F_{\text{in, lin}}^{\text{in}}, \end{aligned} \quad (2)$$

where $\mu = qr$ is the electric dipole moment.

But the third term is very small in comparison to the first and second terms, thus we can neglect the third term. The time-averaged radiation force due to the inhomogeneous force and Lorentz force can be written as

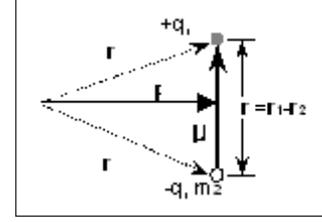


Fig. 2. An electric dipole moment μ whose opposite charges $(+q, -q)$ are separated by distance $|r|$.

$$\langle F \rangle_T = \frac{1}{2} \text{Re} [(\hat{\mu}^* \cdot \nabla) \hat{E} + \hat{\mu}^* \times \hat{B}]. \quad (3)$$

For a complex monochromatic Gaussian beam, the EM fields can be expressed as

$$\begin{aligned} \hat{E} &= \varepsilon(R) \exp(i\phi(R) - i\omega t), \\ \hat{B} &= -\frac{1}{i\omega} \nabla \times \hat{E}, \end{aligned} \quad (4)$$

while the complex-induced complex electric dipole and its time derivative can be written as

$$\hat{\mu} = \alpha \hat{E}. \quad (5)$$

Using vector quantities and Eqs.(4) and (5), we can reexpress Eq. (3) as

$$\langle F \rangle_T = \frac{\alpha' \nabla |\varepsilon(R)|^2}{4} - \frac{\alpha'' |\varepsilon(R)|^2 \nabla \phi}{2}. \quad (6)$$

The first dispersive term corresponds to the dipole force or gradient force, and the second dissipative term is the scattering force. If you consider the Kerr nonlinearity $n_2 = n_2^{(0)} + n_2^{(1)} I(R)$, Eq. (6) becomes

$$\begin{aligned} \langle F \rangle_T &= (2\pi n_1 \alpha^3 / c) \left(\frac{\left(\left[\frac{n_2^{(0)} + n_2^{(1)} I(R)}{n_1} \right]^2 - 1 \right)^2}{\left(\left[\frac{n_2^{(0)} + n_2^{(1)} I(R)}{n_1} \right]^2 + 2 \right)} \right) \nabla I(R) \\ &+ (8\pi n_1 / c) (|k| \alpha)^4 \alpha^2 \left(\frac{\left(\left[\frac{n_2^{(0)} + n_2^{(1)} I(R)}{n_1} \right]^2 - 1 \right)^2}{\left(\left[\frac{n_2^{(0)} + n_2^{(1)} I(R)}{n_1} \right]^2 + 2 \right)} \right) I(R) \end{aligned}$$

where $I(R) \equiv |E(R)|^2$ is the intensity of the Gaussian beam at R of Fig. 1. Notice that the first component is dependent on the gradient of the intensity, while the second term is intensity dependent. The first term points

to the direction of increasing spatial intensity, while the second term points to the direction of the intensity spread or scattering of the incident Gaussian beam on the Kerr nanosphere.

RESULTS AND DISCUSSION

Figure 3 shows how the optical trapping force could generate a pulling action or a pushing action on the Kerr nanosphere. When gradient force or trapping force pulls a nanosphere either $\langle Fz \rangle$ is negative at $z > 0$ or $\langle Fz \rangle$ is positive at $z < 0$. For other cases the nanosphere is pushed away from the beam focus at $z = 0$. It is shown that Kerr nonlinearity increases the magnitude of the trapping force for all pulling situations. Quantitatively, a tenfold increase between the maximum trapping force of a Kerr nanosphere and the maximum trapping force on a linear nanosphere is evident from the figure. Unlike the previous plot, Fig. 4 demonstrates how stiff the trapping system is for both linear and nonlinear nanospheres. As illustrated in Fig. 4, the trapping stiffness for the Kerr nanosphere is much larger than the linear nanosphere which indicates a deeper trapping mechanism.

Figures 3 and 4 are consistent trapping force profiles for both the geometrical ray computation (Pobre & Saloma, 1997) and the Mie computation (Pobre & Saloma, 2002) but at varying orders of magnitude.

However, unlike previous works (Pobre & Saloma, 1997; Pobre & Saloma, 2002), the result in Fig. 5 shows a decrease in the trapping stiffness as the particle size

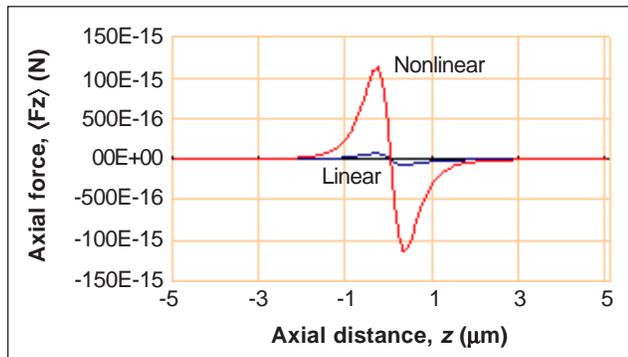


Fig. 3. Axial force as a function of z in (μm). Parameters: $z_0=0$, $r=0$, $p=100$ mW, $a=5$ nm, $\text{NA}=1.2$, $\lambda = 1.064$ μm , $n_1=1.33$, $n_2^{(0)}=1.4$, and $n_2^{(1)}=1.8 \times 10^{-11}$ m^2/W .

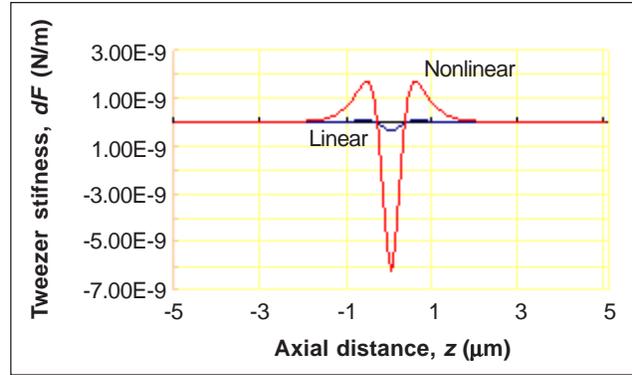


Fig. 4. Stiffness of the trap (dF/dz) as a function of z . Same parameters as in Fig. 3.

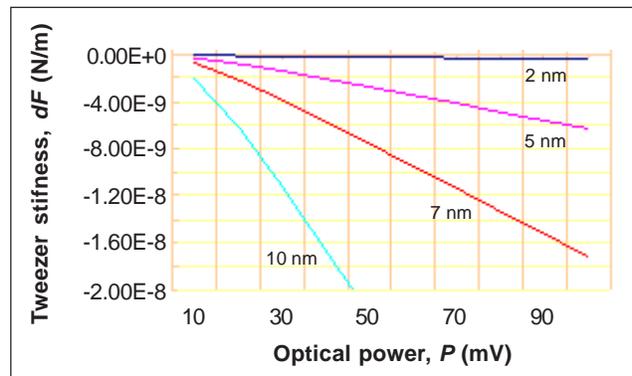


Fig. 5. Tweezer stiffness versus power at different particle sizes with the same parameters as before.

is reduced from 10 to 2 nm. Smaller particles both in the geometrical ray regime and Mie regime result in an increase in the trapping force which points to larger trapping stiffness.

The last figure is a valuable data to experimentalists who work in a tight budget under an optical trapping system. Instead of using high numerical aperture (NA) objective lens (an expensive optics), Fig. 6 would show that it is feasible to trap a Kerr nanosphere even at low NA. Even at high NA (e.g., 1.2) for the linear nanosphere, a low NA (e.g., 0.5) can match the amount of trapping force under the nonlinear counterpart given the same optical parameters.

CONCLUSION

The optical trapping force consists of two governing components. These are the gradient force component

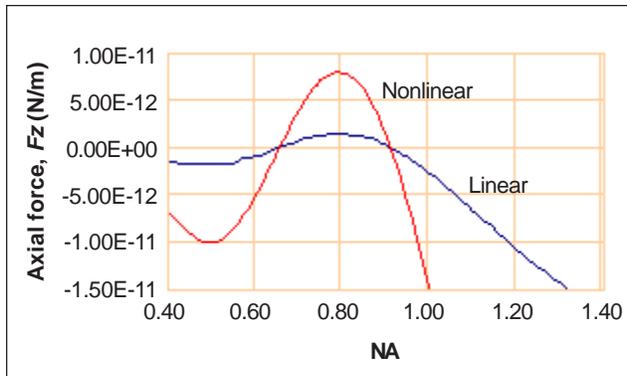


Fig. 6. Axial force as a function of NA with the same optical parameters as before.

(the term that depends on the gradient of the intensity) and the scattering force component. The mechanism of the gradient force stems from two particular mechanisms. The inhomogeneous field force serves only the dipole or gradient force, whereas the Lorentz force contributes both the gradient and scattering forces. Thus, the mechanism of optical trapping is an interplay between the inhomogeneous field force and some component of the Lorentz force when the trapping particle is within the Rayleigh regime. Calculations have shown that both trapping force and trapping stiffness are enhanced with Kerr nonlinearity. Two significant results can be derived from the study. First is the dispersive-like behavior of the trapping force for a Kerr nanosphere, which means that by proper tuning of the incident Gaussian beam you can control the amount of trapping force for nanosized Kerr particles. Second and last interesting observation is its ability to trap even at low NA of the objective lens for Kerr nanospheres. The theoretical model could be extended for the case when it is dependent on the polarization of the incident Gaussian beam. It could be added that size validity between the Mie regime and Rayleigh regime is still an unexplored area of investigation in the size range of 0.1–1 μm .

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