PERT/CPM

PROGRAM EVALUATION REVIEW TECHNIQUE/CRITICAL PATH METHOD

By

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Introduction

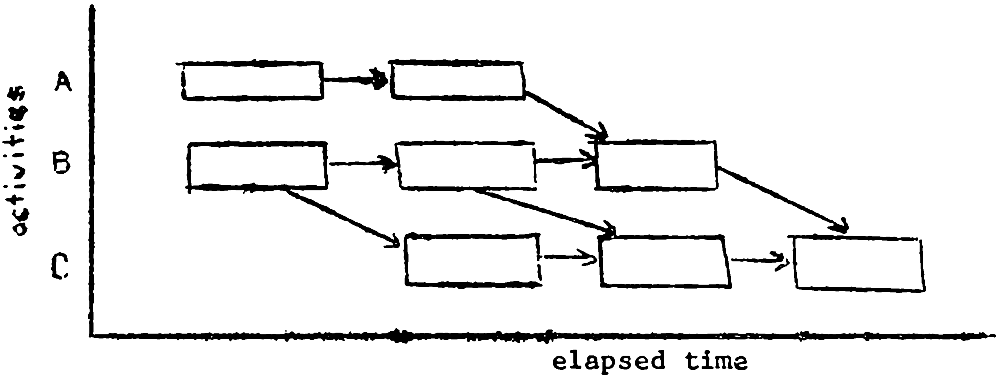
A project consists of a number, usually a large number, of interdependent component tasks or activities. Some of these activities must be done in sequence while others can be done simultaneously. The duration of an activity depends on the amount of resources and therefore on the expenditure allocated to it. The project duration can therefore vary: and how it varies will depend on how the component activities are done and on the amount expended for each activity. This handout deals specifically with a method (PERT/ CPM) of scheduling project activities such that the project is completed in the shortest possible time. It also touches on the planning and controlling functions of the method and considers problems involving one or a combination of the following factors: time, resources and money. The method is introduced as an extenison of a widely used classical method. (Gantt Chart).

Gantt Chart

Among the classical methods used for planning and scheduling, one of the most popular is the use of the so-called Gantt Chart (named after Henry Gantt who originated the method). This simple graphical model shows for each of the compound activities comprising the project the time period over which the activity extends, and the time at which each activity should start in order that each prerequisite activity will be completed or have sufficiently progressed so that the following activities can start.

Although it provides a means of checking the progress of a project, the Gantt Chart has two rather serious limitations; first, it does not clearly indicate the details concerning the progress of activities and; second, it does not clearly indicate what portions of any activity are specifically prerequisite to following activities, or to dependent activities which may overlap.

These deficiences should be eliminated by breaking down the activities into subparts and showing the interdependencies by means of interconnecting arrows as shown below.



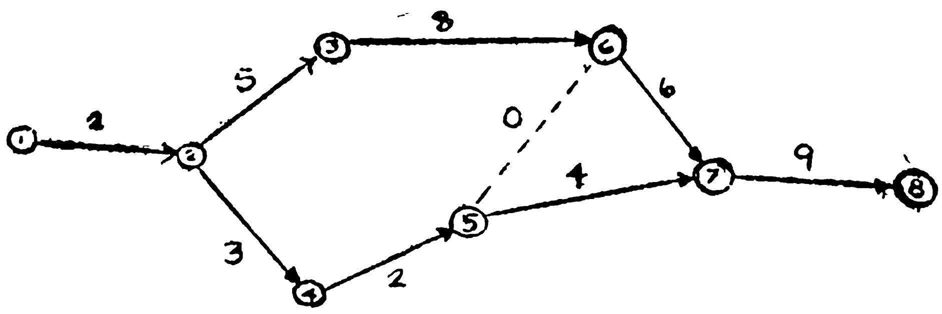
activity — elapsed — time chart, with interdependency shown by arrows.

This leads directly to *networks* which is used in PERT/CPM.

Activity-Event Networks

An activity-event network portrays the interrelationships between the activities and events that comprise a project or a job. (An event is simply a point of transition marking the end of one or more activities and the start of a new group of activities).

There are two systems of representing networks. The first is the activities-on-arrow or AOA system while the second is the activities-on-nodes or AON system where the activities are graphically represented by nodes and the precedence relationships are represented by arrows. The second is not widely used, however, and will not be used extensively in this handout.



The above figure is a typical project network represented by the AOA convention. Events are identified by the numbers in the nodes. The activities are referred to by showing the events that they connect. The above network shows seven events and eight activities. (The activity with zero duration is known as a dummy activity and shall be discussed shortly). The precedence relationships are evident from the network or arrow diagram. Activity (1, 2,) precedes activities (2, 4,), (2,3) and (4, 5) meaning none of the latter three can begin until after activity (1, 2) is completed, etc.

There are several situations that require the use of dummy activities:

1. In the above network, dummy activity (5, 6) is needed to show the dependence of activity (6, 7) on (4, 5).
2. In figure 3 are two parallel activities originating at node 1 and ending at node 2. Note that both activities are labeled (1,2). To distinguish between the two activities, a dummy activity as shown in Figure 3b is added. The two activities could have been combined into one but this may destroy the detail needed in the network.

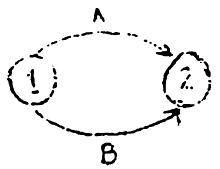


FIGURE 3a

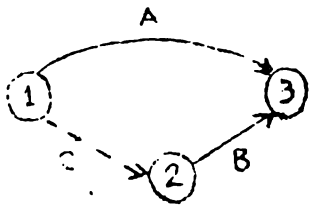


FIGURE 3b

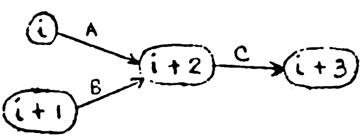


FIGURE 3c

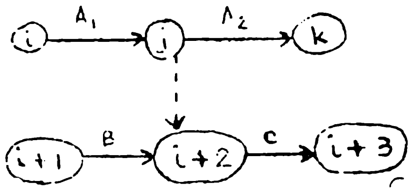


FIGURE 3d

3. In figure 3c, suppose that activity C is only partially dependent on A. Activity A must then be split into two parts, say A1 and A2, such that C, depends on A1 and represent this dependence by a dummy activity as shown in figure 3d.

As a general rule to network construction, Moder and Philips have given the following rules:

1. No activity can begin until all activities preceding it are complete.
2. Neither the length nor the direction of an arc has any significance. It only implies precedence.

3. Two events can be directly connected by at most one activity.

4. Each event number must be unique.

5. A network may have only one initial and one terminal event.

The construction of the network diagram is the planning cycle of PERT/CPM. Many feel that the greatest benefits are derived in this cycle for the project plan must first be will developed and thoroughly analyzed before its network diagram can be constructed.

From the basic rules for constructing network diagrams can be deduced the characteristics or limitations of projects that can benefit from critical analysis. These are:

1. The project consists of a well-defined collection of activities which must be completed to end the project.

2. The activities within a given sequence may be started and stopped independently of each other.

3. The activities must be performed in a given sequence.

Project Scheduling

The next phase after project planning is project scheduling.

The quantities involved in scheduling will be denoted as follows:

t_{ij} = estimated duration time for activity (i, j)

E_j = expected or earliest occurrence time for event i

L_j = latest occurrence time for event i

ES = earliest start for an activity

EF = earliest finish for an activity

LS = latest allowable activity start time

LF = latest allowable activity finish time

S = total activity slack or float

S_F = free slack or float

These quantities will appear on the network diagram in the following manner:

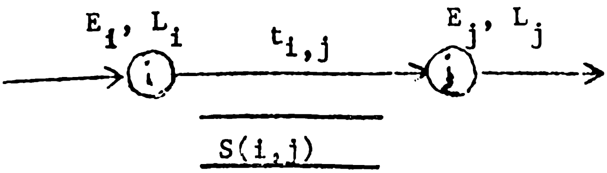


FIGURE 4

The procedure for computing these quantities and determining the so-called critical path will now be illustrated.

The first stage in the scheduling process is to calculate the earliest or expected event times. This calculation is performed by:

1. taking the initial event time of the network as zero: $E_{\bar{1}} = 0$
2. starting each activity as soon as its predecessor event occurs: ES $(i, j) = E_j$
3. determining event times by the largest of the earliest finish times of activities leading to an event. $E_j = \max\ (E_{j2} + t_{j2},)$ where il, ^{x}i2, etc. are preceding events of activities that terminate at event i

The earliest event times will be determined for the network shown in figure 5, first, by using the Gantt Chart and, second, by using the above computational procedure.

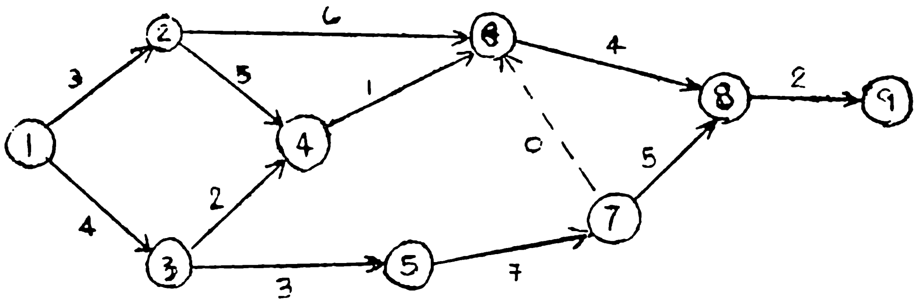


FIGURE 5

The Gantt Chart with all activities at earliest start times can be obtained shifting the activity bars as far to the left as possible without violating any of the precedence relations. Thus, for the network shown in figure 5 Gantt Chart will be as in figure 6.

It can be seen that the earliest occurrence time for event 2 is at $t = 3$, for event 3 at $t = 4$, for event 4 at $t = 8$, for event 5 at $t = 7$, for event 6 at $t = 14$, for event 7 at $t = 14$, for event 8 at $t = 19$ and for event 9 at $t = 21$.

Applying the computational procedure, the earliest occurrence times of the events are obtained as follows:

$E_1 = 0$

$E_2 = E_1 + t_{12} = 0 + 3 = 3$

$E_3 = E_{\bar{1}} + t_{13} = 0 + 4 = 4$

$E_4 = \max\ (E_2 + t_{24},\ E_3 + t_{34}) = \max\ (3 + 5,\ 4 + 2) = 8$

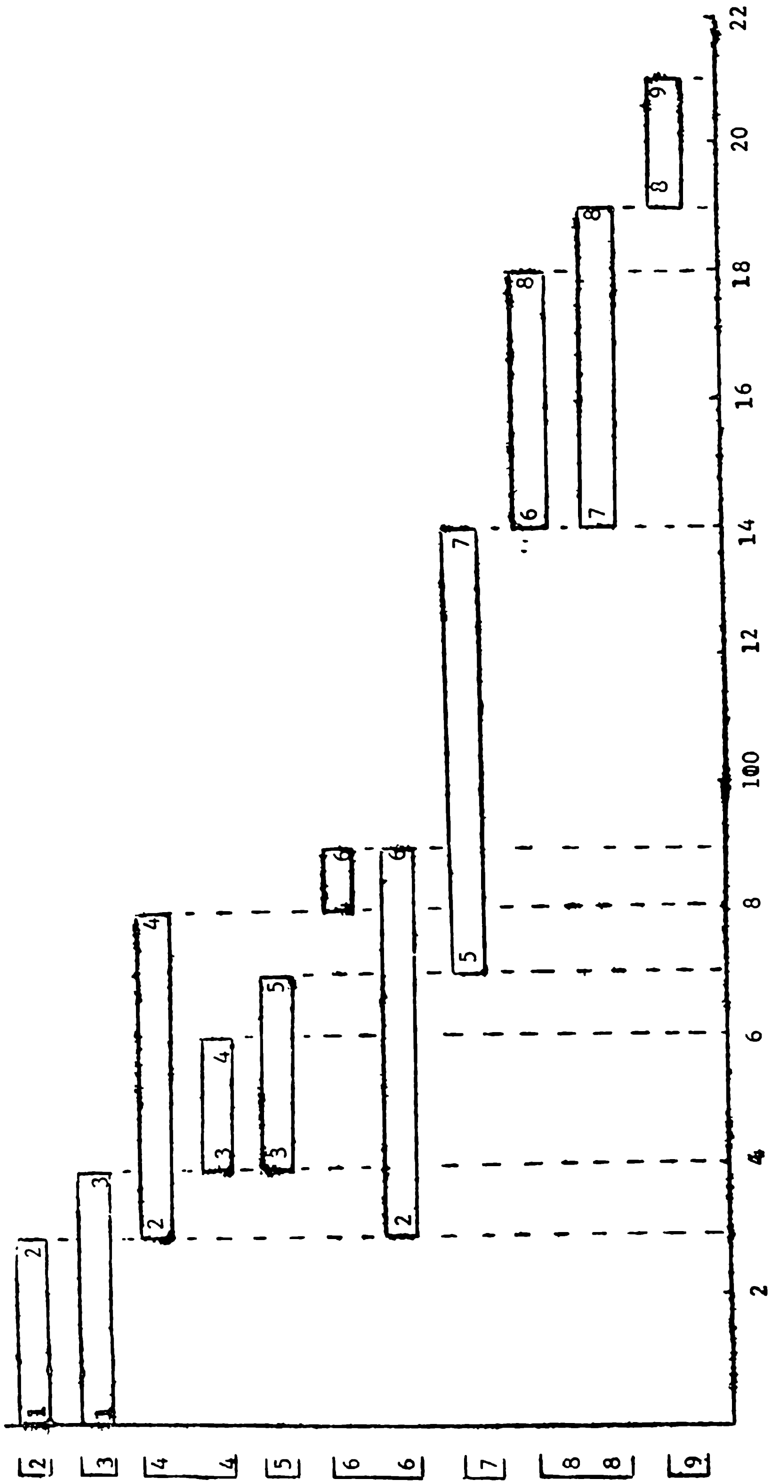


Figure 6

$E_5 = E_3 + t_{35} = 4 + 3 = 7$
$E_6 = \max (E_4 + t_{46}, E_7 + t_{76}) = \max (8 + 1, 14 + 0) = 14$
$E_7 = E_5 + t_{57} = 7 + 7 = 14$
$E_8 = \max (E_6 t_{63}, E_7 + t_{78}) = \max (14 + 4, 14 + 5) = 19$
$E_9 = E_8 + t_{89} = 19 + 2 = 21$

Assuming that the time unit is in weeks, the earliest possible completion time is therefore 21 weeks.

The second stage of the scheduling process is to calculate the latest event times. This calculation is performed by:

1. setting the latest time for the terminal event equal to its earliest time as computed in the forward computation: $L_n = E_n$ where n = terminal event
2. starting each activity at the latest time of its successor event less the duration time of the activity: LS (i, j) $= L_j - t_{ij}$ or LS (i, j) = LF (i, j)$-t_{ij}$ since LF (i, j) $= L_j$
3. determining event times by the smallest of the latest allowable start times of all activities emanating from an event: $L_j = \text{mix} \quad (L_{jj} = t_j \quad L_{j2} - t_j,$ j2,......) where ji, j2, j3,...... are all successor events of activities which start at event i.

The Gantt Chart and the computational procedure will again be used to determine the event times (latest) in this second stage of the scheduling process. For the sample network, the Gantt Chart with all activities at latest start times can be obtained by setting the latest time for event 9.

(the terminal event to t = 21 (its earliest occurrence time) and then shifting all activity bars as far to the right as possible without violating any of the precedence relations. The result is shown in figure 7.

Applying the computational procedure, the latest occurrence times of the events are obtained as follows:

$L_9 = E_9 = 21$
$L_8 = L_9 - t_{89} = 21 - 2 = 19$
$L_6 = L_8 - t_{68} = 19 - 4 = 15$
$L_7 = \min (L_6 - t_{76}, L_8 - t_{78}) = \min (15, 19-5) = 14$
$L_5 = L_7 - t_{57} = 7$
$L_4 = L_6 - t_{46} = 14$
$L_3 = \min (L_5 - t_{35}, L_4 - t_{34}) = \min (4, 12) = 4$
$L_2 = \min (L_6 - t_{26}, L_4 - t_{24}) = \min (9,9) = 9$
$L_1 = \min (L_2 - t_{12}, L_3 - t_{13}) = \min (6,0) = 0$

The third stage of the scheduling process is to determine the slack times (total and free slacks) for the activities.

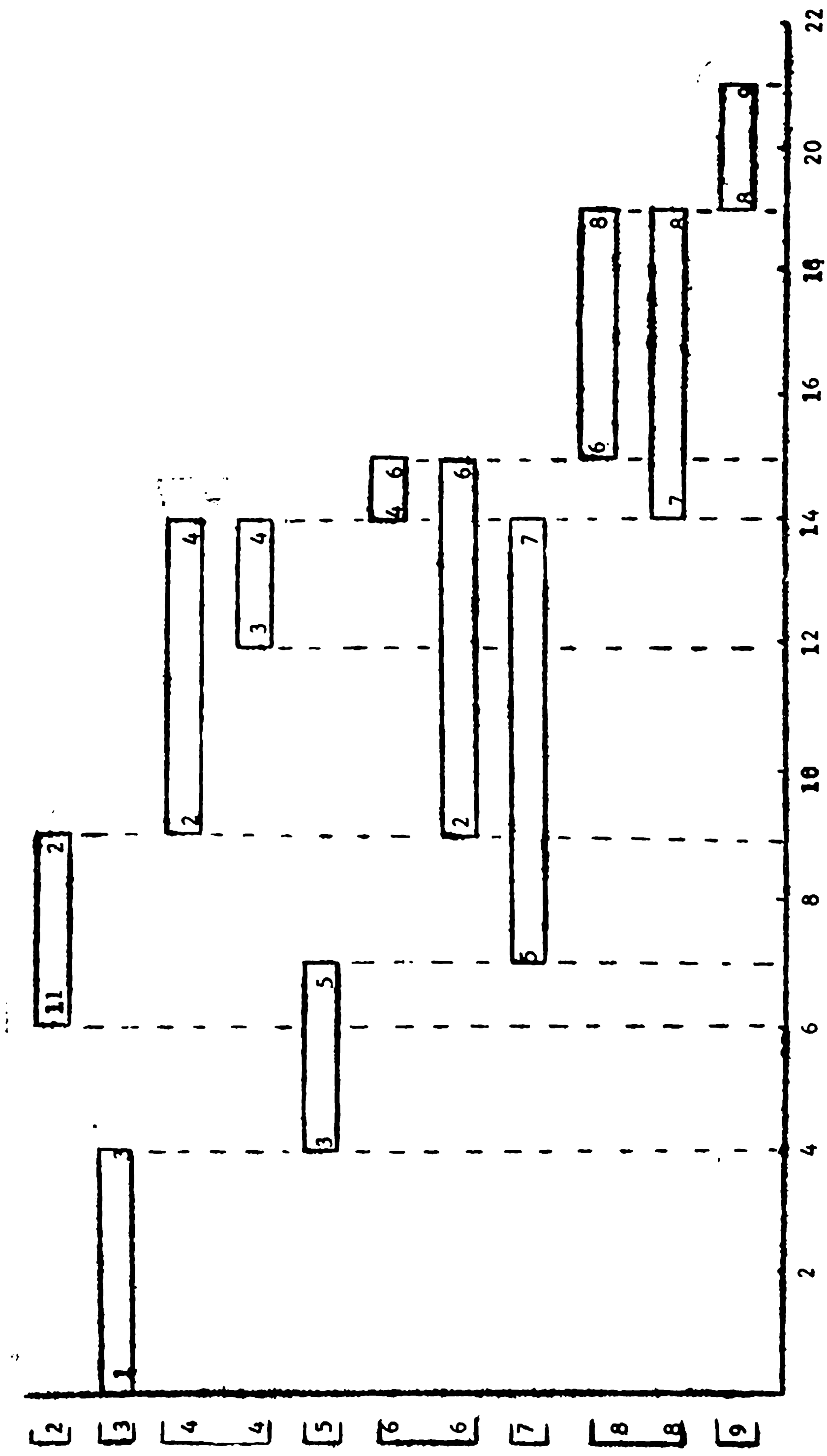


Figure 7

To obtain the total slack for activity (i, j), first, schedule the activities at their earliest start times and, second, at their latest start times. The total slack of activity (i, j) will then be distance in the time axis by which the activity bar of (i, j) is moved in shifting it from its earliest start position to its latest start position. For instance, the total slack of (8, 9) is zero since its earliest start position coincides with its latest start position. The total slack of (6, 8) is one since the activity bar corresponding to (6, 8) is displaced by one time unit in shifting it from its earliest start position to its latest start position. And so on.

From its graphical illustration, it can be seen that the total slack for an activity measures that time by which that activity could be extended or displaced without affecting the total project completion date. It can be computed as follows:

$$S(i,j) = L_j - EF\ (i,j)$$
$$= L_j - (E\ + tij) \text{ since } EF\ (i, j) = (E + tij)$$
$$= LF\ (i, j) - EF)i, j) \text{ since } L_j = LF\ (i, j)$$

The total slacks for the activities in the sample network can be obtained as follows:

$$S(8, 9) = L_9 = (E_8 + t_{89}) = 21 - (19 + 2) = 0$$
$$S(6, 8) = L_8 = (E_6 + t_{68}) = 19 - (14 + 4) = 1$$
$$S(7, 8) = L_8 - (E_7 + t_{78}) = 19 - (14 + 5) = 0$$
$$S(7, 6) = L_6 - (E_7 + t_{76}) = 15 - (14 + 0) = 1$$
$$S(5,7) = L_7 - (E_5 + t_{57}) = 14 - (7 + 7) = 0$$
$$S(4,6) = L_6 - (E_4 + t_{46}) = 15 - (8 + 1) = 6$$
$$S(2,6) = L_6 - (E_2 + f_{26} = 15 - (3 + 6) = 6$$
$$S(2,4) = L_4 - (E_2 + t_{24}) = 14 - (3 + 5) = 6$$
$$S(3,5) = L_5 - (E_3 + t_{35}) = 7 - (4 + 3) = 0$$
$$S(3,4) = L_4 - (E_3 + t_{34}) = 14 - (3 + 5) = 6$$
$$S(1,3) = L_3 - (E_1 + t_{13}) = 4 - (0 + 4) = 0$$
$$S(1,2) = L_2 - (E_1 + t_{12}) = 9 - (0 + 3) = 6$$

The Free float or Free slack of an activity (i, j) is the amount of time by which its completion time can be delayed without affecting the earliest start time for any other activity in the network. It follows from this definition that the Free slack of an activity is always equal or less than its total slack and is computed as follows:

$$SF\ (i,j) = E_j - EF\ (i,j)$$
$$= E_j - (E_j + t_{i,j})$$

For the sample network, they are obtained as follows:
Since $S_f\ (i,j) \leq S\ (i,j)$, it follows that

$$S_F(8,9)\ {}_F= S\ (7,8) = S_F\ (5,7) = S_F(1,3) = S_F(3,5) = 0$$

$$S_F(6,8) = E_8 - (E_6 + t_{68}) = 19 - (14 + 4) = 1$$

$$S_F(7,6) = E_6 - (E_7 + t_{76}) = 14 - (14 + 0) = 0$$

$$S_F(4,6) = E_6 - (E_4 + t_{46}) = 14 - (8 + 1) = 5$$

$$S_F(2,6) = E_6 - (E_2 + t_{26}) = 14 - (3 + 6) = 5$$

$$S_F(2,4) = E_4 - (E_2 + t_{24}) = 8 - (3 + 5) = 0$$

$$S_F(3,4) = E_4 - (E_3 + t_{34}) = 8 - (4 + 2) = 2$$

$$S_F(1,2) = E_2 - (E_1 + t_{12}) = 3 - (0 + 3) = 0$$

At this point, the critical path or, equivalently, the critical activities comprising the critical path can now be determined. The critical activities are those activities with zero slacks. For the sample network these are (8,9), (7,8), (5,7), (3,5) and (1,3). The critical path is therefore (1,3,5,7,8,9). None of the activities along this path can be delayed without delaying the project.

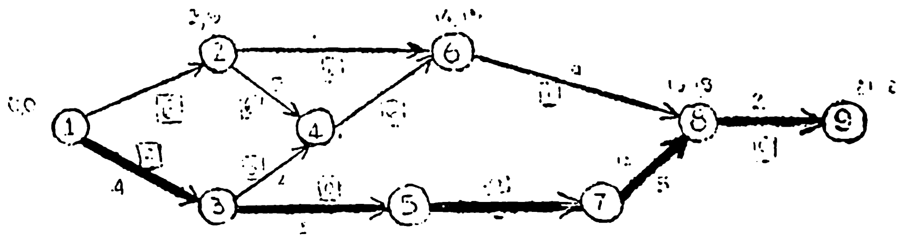


Figure 8

critical activities: (1,3), (3,5), (5,7), (7,8) & (8,9) note that for a critical activity (i,j), (1) $S\ (i,j) = 0$, (2) $E_i = L_i$, (3) $E_j = L_j$ and (4) $E_i = E_j - t_{ij}$

(Please see Figure 4 for the interpretation of the above network).

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